Anomalous Diffusion and Quantum Interference Effect in Nano-scale Periodic Lorentz Gas

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Abstract

Recent advances in submicrometer technology have made it possible to confine the two-dimensional electron gas into high-mobility semiconductor heterostructures. Such structure with a lattice of electron-depleted circular obstacles are called quantum antidot lattices, or quantum Lorentz gas systems. By using the semiclassical scattering theory, we show that quantum interference in finite-size open Lorentz gas systems is expected to reflect the difference between normal and anomalous diffusions, i.e., Lévy flights.

Recent advances in submicrometer technology have made it possible to confine the two-dimensional electron gas into high-mobility semi-conductor heterostructures. Such structure with a lattice of electron-depleted circular obstacles are called *antidot lattices* [1] and can be regarded as nano-scale periodic Lorentz gas.

In the hexagonal lattice Lorentz gas with $R/L < \sqrt{3}/4$ (R and L are the radius of the circle and the width of the unit-cell, respectively), there exist arbitrarily long paths along which classical particles can move freely without touching the hard discs (antidots). Thus, the diffusion in this system becomes anomalous and can be modeled by Lévy flights [2]. In the case of sufficiently large radius compared to the lattice constant, *i.e.*, $R/L > \sqrt{3}/4$, on the other hand, collisionless long trajectories can no longer exist and the diffusion becomes normal. Therefore, we can expect that the quantum interference between electron paths in these systems is expected to reflect the difference between normal and anomalous diffusions. In this paper, we shall investigate the *anomalous diffusion of quantum particles* in finite-size Lorentz gas attached to the lead wires by use of the semi-classical theory.

The quantum-mechanical conductance is related to the transmission amplitude $t_{n,m}$ by the Landauer formula [3],

$$g = \frac{e^2}{\pi\hbar} \sum_{n,m=1}^{N_M} |t_{n,m}|^2 , \tag{1}$$

where N_M is the number of the mode in the lead wire. $t_{n,m}$ is exactly given by a double integral of the retarded Green's function G at the Fermi energy [4],

$$t_{n,m} = c_{n,m} \int dy \int dy' \psi_n^*(y')\psi_m(y)G(y', y, E_F). \tag{2}$$

In eq.(2) $c_{n,m} \equiv i\hbar\sqrt{\upsilon_n\upsilon_m}$, where $\upsilon_m(\upsilon_n)$ is the longitudinal velocity, and $\psi_m(\psi_n)$ is transverse wave function for the mode $m(n)$. To approximate $t_{n,m}$ we replace G by its semi-classical Feynman path-integral expression [5],

$$G^{sc}(y', y, E) = \frac{2\pi}{(2\pi i\hbar)^{3/2}} \sum_{s(y,y')} \sqrt{D_s} \exp\left[\frac{i}{\hbar} S_s(y', y, E) - i\frac{\pi}{2}\mu_s\right], \tag{3}$$

where S_s is the action integral along classical path s, $D_s = (\upsilon_F \cos\theta')^{-1} \left|(\partial\theta/\partial y')_y\right|$, θ (θ') is the incoming (outgoing) angle, and μ_s is the Maslov index. Substituting eq. (3) into eq. (2) and using the dwelling time distribution, we finally obtain the correlation function for the $g(k)$ as

$$\begin{aligned} C(\Delta k) &\equiv \langle \delta g(k) \delta g(k+\Delta k) \rangle_k \\ &= \frac{e^4}{16\pi^2\hbar^2} \frac{1}{1+(l_0\Delta k)^2}, \end{aligned} \tag{4}$$

where $\delta g = g - g_{cl}$ (g_{cl} is the classical conductance) and l_0 is the typical dwelling length in the Lorentz gas [6].

From the classical simulations, we have confirmed that l_0 damps exponentially fast with decreasing R/L. Therefore in the case that $R/L \ll (>) \sqrt{3}/4$, $g(k)$ oscillates regularly(irregularly). This result means that we can experimentally observe the quantum signature of anomalous diffusion in the Lorentz gas through quantum interference effects.

References

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